MARATHA VIDYA PRASARAK SAMAJ'S

KARMAVEER ADV. BABURAO GANPATRAO THAKARE

COLLEGE OF ENGINEERING

KBTCOE

www.kbtcoe.org

Permanently Affiliated to Savitribai Phule Pune University Vide Letter No: CA/1542

& Approved by AICTE New Delhi Vide Letter No: 740-89-32 (E) ET/98

AISHE Code - C-41622

Department of Mechanical Engineering

Innovative Teaching Method - Report

Academic Year –2020-21	Class -SE
Semester–II	Date:05/7/2021
CO:CO1,CO2,CO3,CO4,CO5,CO6	PO:PO1, PO2, PO8, PO12

1. **Title of Innovation method/activity: Mind map** by using Google Classroom
2. Link shared to the students:

   https://classroom.google.com/u/0/w/Mjk0NDE3OTA3MDM2/t/all
3. Name of Faculty: Ms.M.G.Dhumase
4. Course :Engineering Mathematics-III (207002)
5. Objective of Method
   a. Clear the concept.
   b. It helps students to think individually.
   c. More involvement of students.

6. Topic Covered through Activity

Engineering Mathematics-III

7. Description of method with Benefits (8 –10lines)

Description of method

Monitor and support students for performing Activity:

By using this method we are able to check the concept understood by the students. Also students get engaged and show their creativity while preparing Mind Map. Students are able to revise the topic very easily.

Benefits of method

- It helps students to think individually about a topic and clear their concept.
- It helps students to develop their creativity.
- It helps students to understand the concepts and revise the topic.
- Students know the application which increase the interest of students in Mathematics

8. Roles and Responsibilities

- **Teacher**
  - Elaborate regarding activity.
  - Encourage students to prepare Mind Map and upload it on Google Classroom.
  - Provide the study material on Topic.
  - Remain available during the completion of the task.
  - Prepare assessment methodology.
- **Student**
  - Go through the concept of the topic.
  - Understand the concept and show their creativity independently while preparing

the Mind Map and upload the PDF on or before the given time.

- Actively participate in Mind Map activity and contribute their knowledge regarding the topic covered.

9. Assessment Tools: Maximum Marks 25

Involvement , Understanding and performance	10	7	4
	Excellent	Good	poor
	All topics are covered with correct procedure and application.	some topics are covered with correct procedure and application	Few topics are covered with some error in procedure and application is missing
Presentation and Organization	10	7	4
	Excellent	Good	poor
	Neatness is Good and with creativity	Neatness is OK and with less creativity	less Neatness and with no creativity or file of other student is uploaded
Timely submission	5	3	0
	Excellent	Good	poor
	submitted within deadline	late Submitted	not submitted

10. Evaluation sheet of attendee: Separate sheet attached.

Roll No	Name of the Student	Involvement , Understanding & Performance (10)	Presentation& Organization (10)	Timely Submission (5)	Total (25)
81	Malve Utkarsh Narendra	7	7	5	19
82	Malsane Bhushan Shantaram	7	7	3	17
83	Mandal Himanshushekhar Keshav	7	7	4	18
84	Mhaskar Shubham Somnath	7	4	4	15
87	Muthal Sahil Shashikant	7	4	4	15
89	Nishad Vishal Dinesh	7	7	5	19
90	Pachore Pratik Dattatray	7	6	3	16
91	PAGARE ALOK SATISH	7	7	5	19
94	Pingle Rohit Dilip	7	10	3	20
96	Patil Chinmay Lalit	10	10	4	24
97	Patil Dev Sachin	10	4	4	18
99	Patil Pranav Rajendra	10	7	4	21
106	SALUNKE PRATIK RAMESH	8	8	4	20
109	Sarkar Sarthak	7	7	5	19

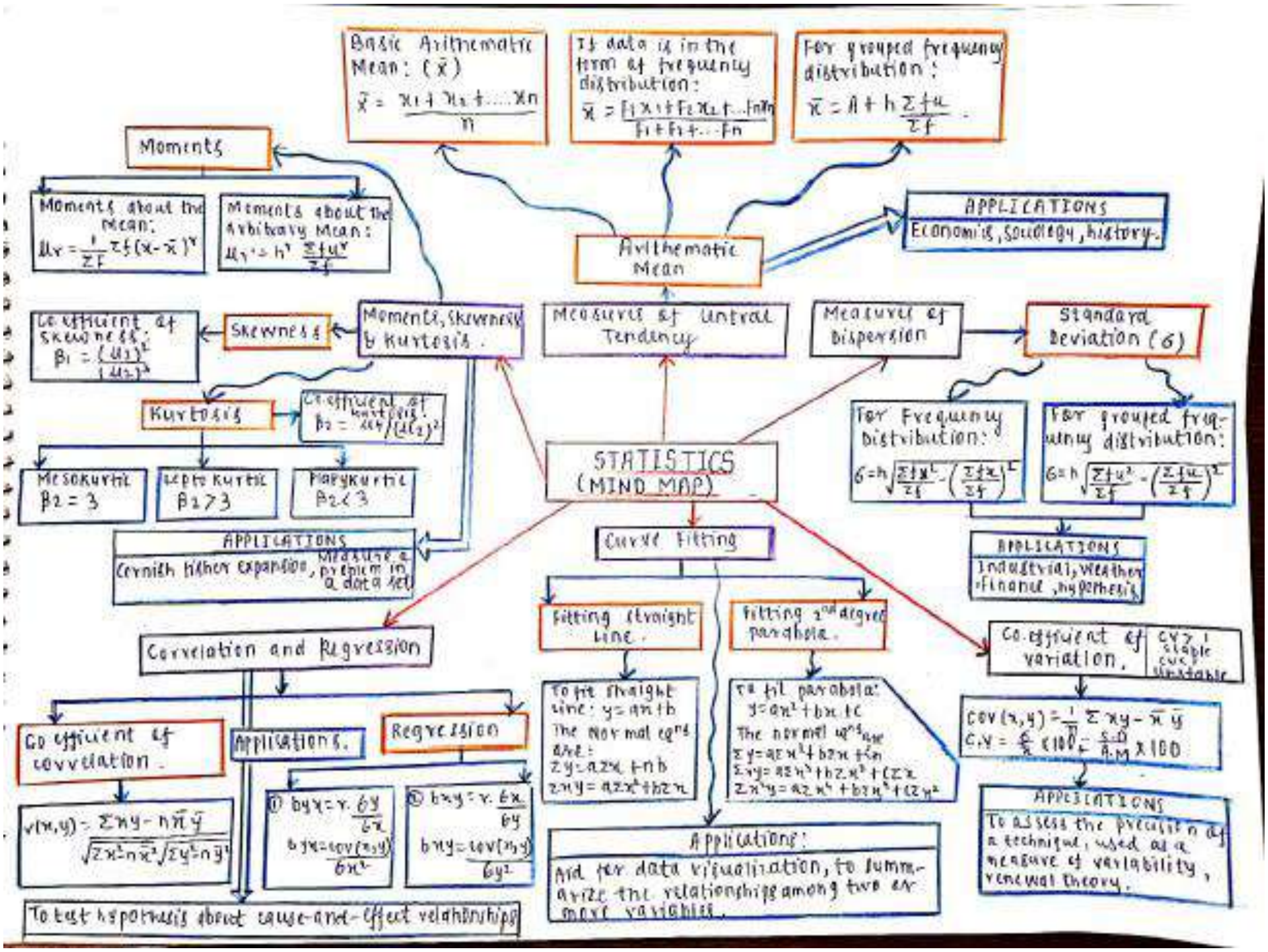
113	Shinde_Mayur_Vasant	7	10	4	21
114	Shinde Yash Bhimrao	7	7	3	17
115	Shirsath Mayur Somnath	7	7	3	17
116	SHIRSATH OMKAR VISHVANATH	7	7	5	19
118	Sonawane Suyash Sharad	10	10	5	25
119	Sonawane Yash Sharad	10	10	5	25
120	Tambe Abhishek Vijay	10	10	5	25
121	TASKAR JAYDATT CHANDRAKANT	10	7	5	22
124	Ugale Tanay Sudhir	7	7	5	19
127	Wagh saurabh sanjay	7	4	3	14
129	Watpade Akshay Balkrishna	10	7	5	22
131	Yadav Deepak Shyamkishor	7	10	5	22
132	Zade Vivek Ramesh	10	10	5	25
133	PATIL NAKSHATRA HEMANT	7	7	5	19
136	Raijade Akshada Sanjay	10	7	5	22
140	Shingade vedant mahesh	7	7	5	19
146	Bharsat Bhushan Pundlik	7	10	4	21
147	Budhvant Dhanajay rajendra	7	7	3	17
148	Daspute Amit Milind	7	7	5	19
150	Gaikwad Shantanu Prakash	7	10	4	21
151	Handore Aditya Dayaram	7	7	3	17
154	Kamble Sujata prakash	7	10	4	21
155	Karmase Aniket Pravin	7	7	4	18
156	Kaklij Ritesh Jibhau	7	7	5	19
157	Kurware Shraddha Ankush	7	10	5	22
158	Modak Ajinkya Vilas	7	10	5	22
161	Patole Rohit Pandurang	10	10	4	24
164	Shinde Sakshi	7	10	5	22
165	SHEWALE SWAYAM TUSHAR	10	10	5	25
168	Vadnere prathamesh suhas	7	10	5	22
169	Sonawane Ankit Ramesh	7	7	5	19

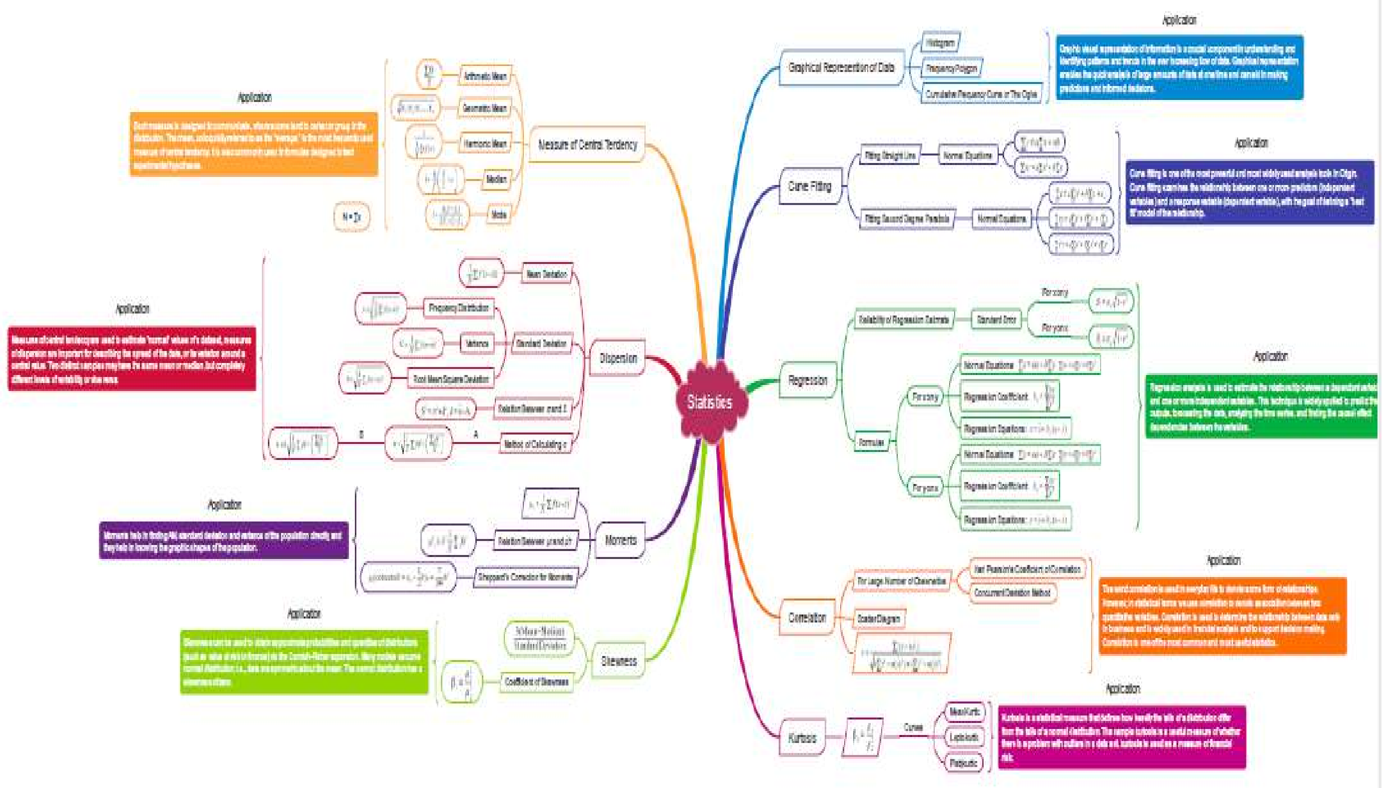
11. Impact Analysis

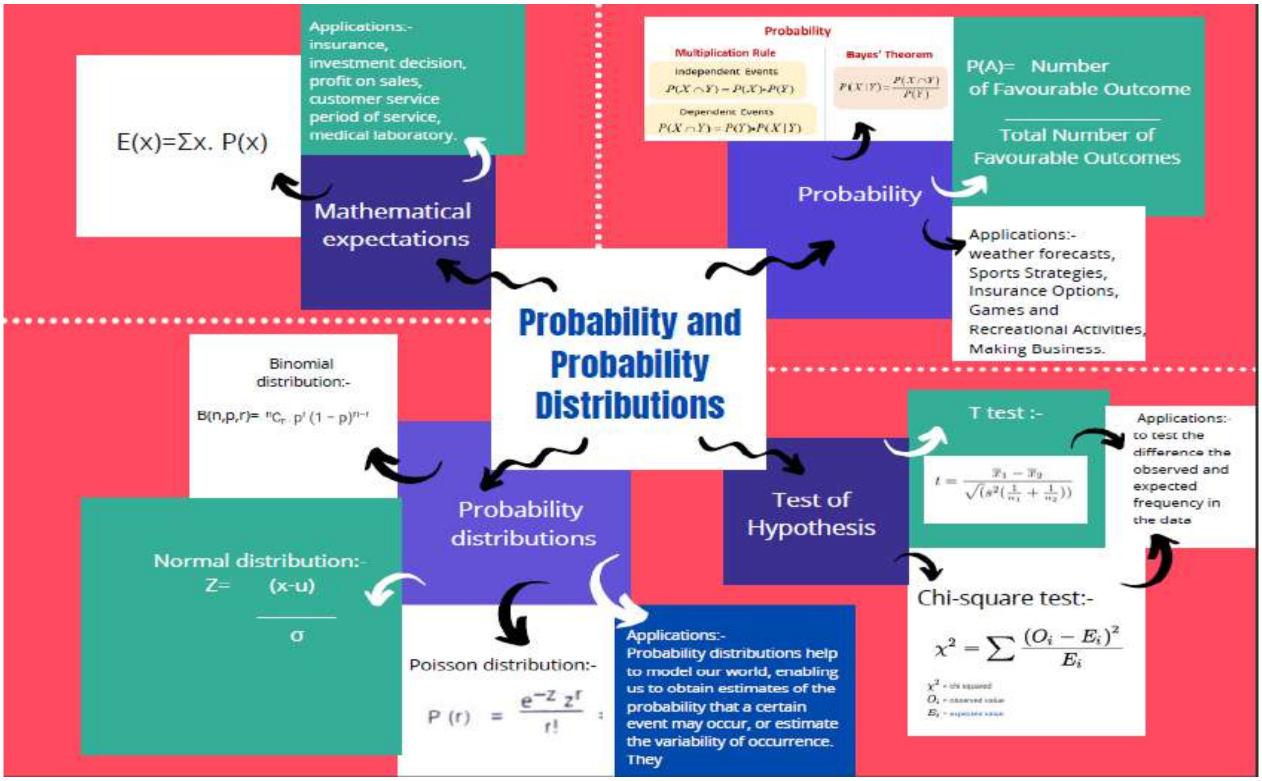
Sr. No.	3-High/Excellent	2-Moderate /Average	1-Slight/Poor
1. Did you understand and cover the objective of the activity?	71.40%	28.60%	-

2. Do you find that methodology is helpful to cover the content beyond syllabus?	73.20%	26.80%	-
3.Does this help you increase your knowledge of the topic?	75%	25%	-
4. Does the content covered are relevant and will be helpful as a Life-long learning?	64.30%	28.60%	7.10%
5. Do you want to conduct such an activity again?	73.20%	25%	1.80%

12. Activity Picture







Probability and Probability Distributions

Mathematical expectations

E(x)=Σx. P(x)

Applications:-
insurance,
investment decision,
profit on sales,
customer service
period of service,
medical laboratory.

Probability

Probability

Multiplication Rule

Independent Events

$P(X \cap Y) = P(X) \cdot P(Y)$

Dependent Events

$P(X \cap Y) = P(Y) \cdot P(X \mid Y)$

Bayes' Theorem

$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$

P(A)= Number of Favourable Outcome / Total Number of Favourable Outcomes

Applications:-
weather forecasts,
Sports Strategies,
Insurance Options,
Games and
Recreational Activities,
Making Business.

Probability distributions

Binomial distribution:-

B(n,p,r)= ${}^{n}C_{r} \cdot p^{r}(1-p)^{n-r}$

Normal distribution:-
Z= (x-u) / σ

Poisson distribution:-

P (r) = $\frac{e^{-z} z^{r}}{r!}$:

Applications:-
Probability distributions help to model our world, enabling us to obtain estimates of the probability that a certain event may occur, or estimate the variability of occurrence. They

Test of Hypothesis

T test :-

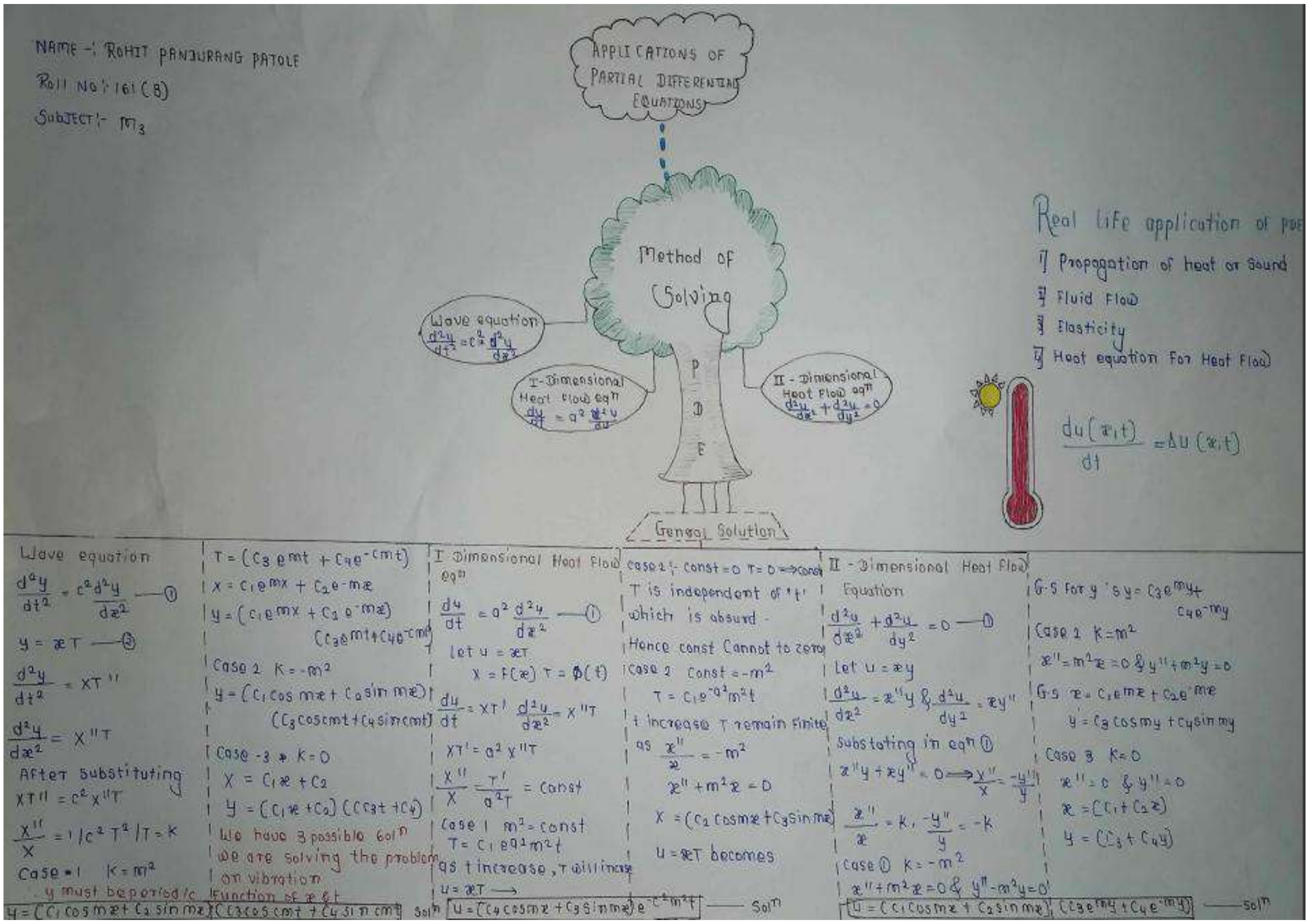
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s^2(\frac{1}{n_1} + \frac{1}{n_2}))}}$$

Applications:-
to test the difference the observed and expected frequency in the data

Chi-square test:-

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

χ^2 = chi squared
O_i = observed value
E_i = expected value



NAME -: ROHIT PANDURANG PATOLE
Roll No :- 161 (B)
SUBJECT :- M_3

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Real life application of PDE

1) Propagation of heat or sound
2) Fluid Flow
3) Elasticity
4) Heat equation For Heat Flow

$$\frac{du(x,t)}{dt}=\Delta u(x,t)$$

Wave equation

$\frac{d^2y}{dt^2}=c^2\frac{d^2y}{dx^2}$ —— ①

$y=XT$ —— ②

$\frac{d^2y}{dt^2}=XT''$

$\frac{d^2y}{dx^2}=X''T$

After substituting

$XT''=c^2X''T$

$\frac{X''}{X}=1/c^2\,T''/T=k$

Case - 1 $k=m^2$

$T=(c_3e^{cmt}+c_4e^{-cmt})$

$X=c_1e^{mx}+c_2e^{-mx}$

$y=(c_1e^{mx}+c_2e^{-mx})(c_3e^{cmt}+c_4e^{-cmt})$

Case 2 $k=-m^2$

$y=(c_1\cos mx+c_2\sin mx)(c_3\cos cmt+c_4\sin cmt)$

Case - 3 $k=0$

$X=c_1x+c_2$

$y=(c_1x+c_2)(c_3t+c_4)$

We have 3 possible soln

we are solving the problem on vibration

∴ y must be periodic function of x & t

$y=(c_1\cos mx+c_2\sin mx)(c_3\cos cmt+c_4\sin cmt)$ soln

I Dimensional Heat Flow eqn

$\frac{du}{dt}=a^2\frac{d^2u}{dx^2}$ —— ①

let $u=XT$

$X=f(x)$ $T=\phi(t)$

$\frac{du}{dt}=XT'$ $\frac{d^2u}{dx^2}=X''T$

$XT'=a^2X''T$

$\frac{X''}{X}=\frac{T'}{a^2T}=\text{const}$

Case 1 $m^2=\text{const}$

$T=c_1e^{a^2m^2t}$

as t increase, T will increase

Case 2 :- const = 0 T = 0 ⟹ const

T is independent of 't' which is absurd.

Hence const Cannot to zero

Case 3 Const $=-m^2$

$T=c_1e^{-a^2m^2t}$

t increase T remain finite

as $\frac{X''}{X}=-m^2$

$X''+m^2X=0$

$X=(c_2\cos mx+c_3\sin mx)$

$u=XT$ becomes

$u=XT\longrightarrow$

$u=(c_4\cos mx+c_3\sin mx)e^{-c^2m^2t}$ —— soln

II - Dimensional Heat Flow Equation

$\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}=0$ —— ①

Let $u=xy$

$\frac{d^2u}{dx^2}=x''y$ & $\frac{d^2u}{dy^2}=xy''$

Substituting in eqn ①

$x''y+xy''=0\Longrightarrow\frac{x''}{x}=\frac{-y''}{y}$

$\frac{x''}{x}=k,\ \frac{-y''}{y}=-k$

case ① $k=-m^2$

$x''+m^2x=0$ & $y''-m^2y=0$

G.S for y 's $y=c_3e^{my}+c_4e^{-my}$

Case 2 $k=m^2$

$x''-m^2x=0$ & $y''+m^2y=0$

G.S $x=c_1e^{mx}+c_2e^{-mx}$

$y=c_3\cos my+c_4\sin my$

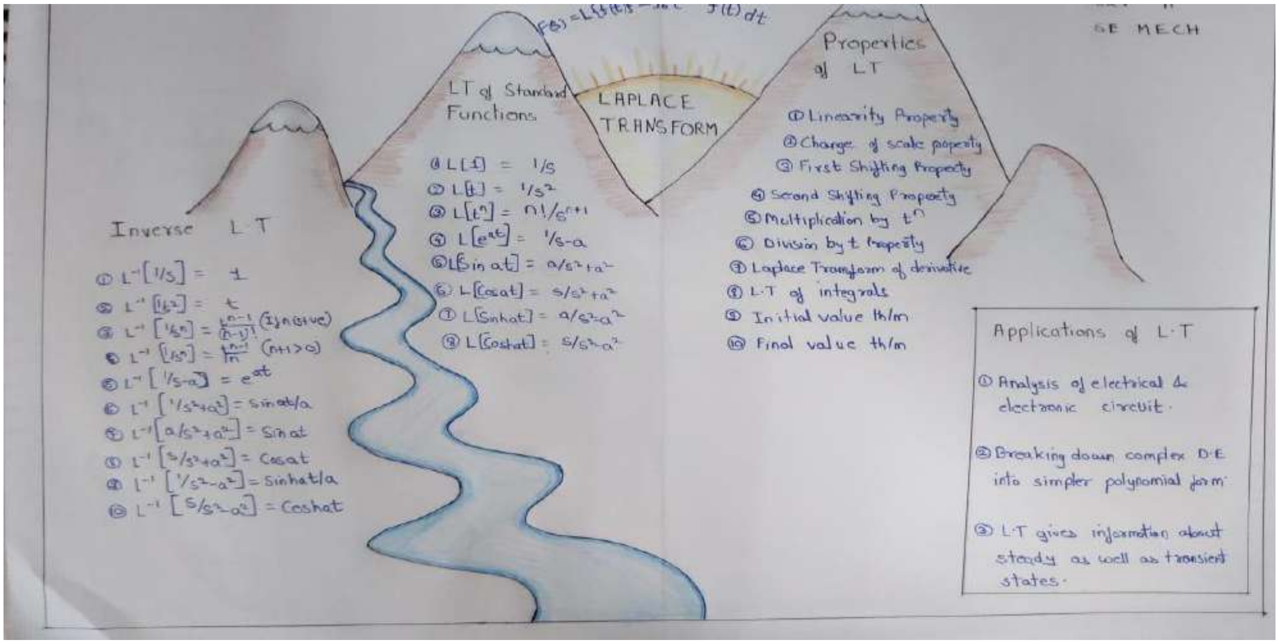
Case 3 $k=0$

$x''=0$ & $y''=0$

$x=(c_1+c_2x)$

$y=(c_3+c_4y)$

$u=(c_1\cos mx+c_2\sin mx)(c_3e^{my}+c_4e^{-my})$ —— soln



SE MECH

LAPLACE TRANSFORM

$F(s)=L\{f(t)\}=\int e^{-st}f(t)\,dt$

LT of Standard Functions

① $L[1]=1/s$
② $L[t]=1/s^2$
③ $L[t^n]=n!/s^{n+1}$
④ $L[e^{at}]=1/s-a$
⑤ $L[\sin at]=a/s^2+a^2$
⑥ $L[\cos at]=s/s^2+a^2$
⑦ $L[\sinh at]=a/s^2-a^2$
⑧ $L[\cosh at]=s/s^2-a^2$

Properties of LT

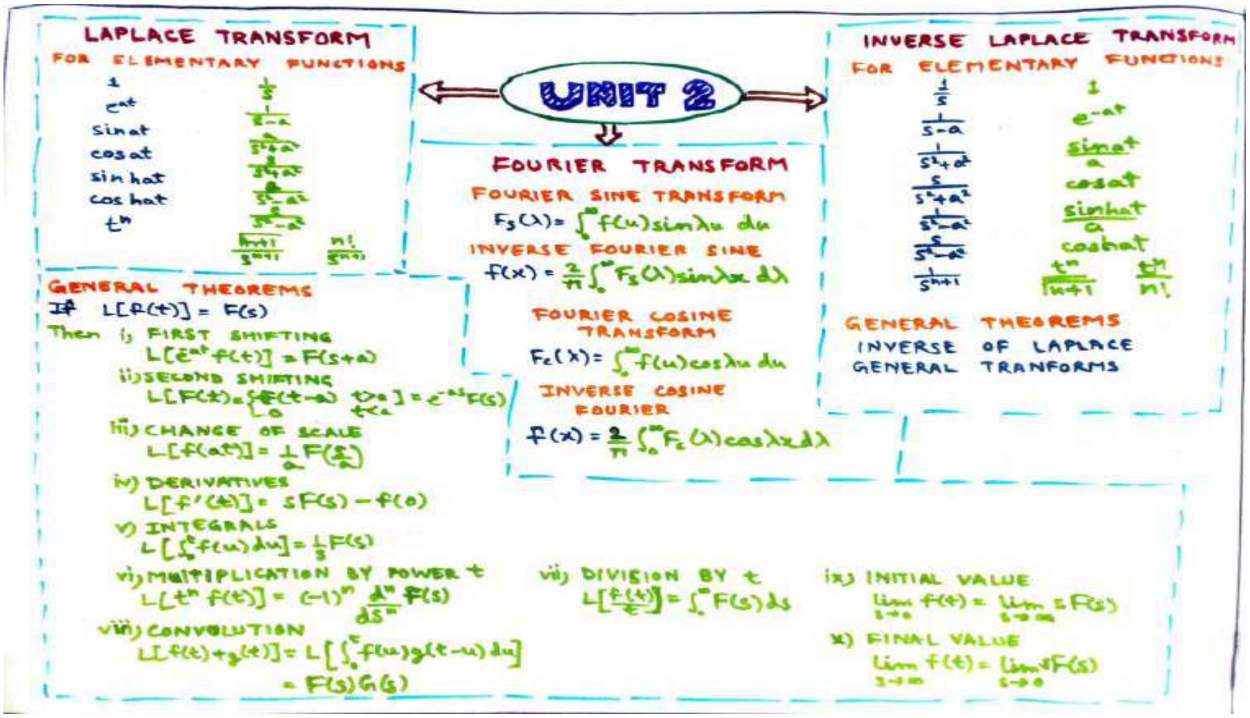
① Linearity Property
② Change of scale property
③ First Shifting Property
④ Second Shifting Property
⑤ Multiplication by t^n
⑥ Division by t Property
⑦ Laplace Transform of derivative
⑧ L.T of integrals
⑨ Initial value th/m
⑩ Final value th/m

Inverse L T

① $L^{-1}[1/s]=1$
② $L^{-1}[1/s^2]=t$
③ $L^{-1}[1/s^n]=\frac{t^{n-1}}{(n-1)!}$ (If n is +ve)
④ $L^{-1}[1/s^n]=\frac{t^{n-1}}{\Gamma n}$ $(n+1>0)$
⑤ $L^{-1}[1/s-a]=e^{at}$
⑥ $L^{-1}[1/s^2+a^2]=\sin at/a$
⑦ $L^{-1}[a/s^2+a^2]=\sin at$
⑧ $L^{-1}[s/s^2+a^2]=\cos at$
⑨ $L^{-1}[1/s^2-a^2]=\sinh at/a$
⑩ $L^{-1}[s/s^2-a^2]=\cosh at$

Applications of L.T

① Analysis of electrical & electronic circuit.
② Breaking down complex D.E into simpler polynomial form.
③ L.T gives information about steady as well as transient states.



UNIT 2

LAPLACE TRANSFORM

FOR ELEMENTARY FUNCTIONS

1	$\frac{1}{s}$	
e^{at}	$\frac{1}{s-a}$	
$\sin at$	$\frac{a}{s^2+a^2}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sinh at$	$\frac{a}{s^2-a^2}$	
$\cosh at$	$\frac{s}{s^2-a^2}$	
t^n	$\frac{\lceil n+1}{s^{n+1}}$	$\frac{n!}{s^{n+1}}$

GENERAL THEOREMS

If $L[f(t)] = F(s)$

Then i) FIRST SHIFTING

$L[e^{-at}f(t)] = F(s+a)$

ii) SECOND SHIFTING

$L\left[F(t) = \begin{cases} f(t-a) & t>a \\ 0 & t<a \end{cases}\right] = e^{-as}F(s)$

iii) CHANGE OF SCALE

$L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

iv) DERIVATIVES

$L[f'(t)] = sF(s) - f(0)$

v) INTEGRALS

$L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}F(s)$

vi) MULTIPLICATION BY POWER t

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s)$

vii) DIVISION BY t

$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

viii) CONVOLUTION

$L[f(t)*g(t)] = L\left[\int_0^t f(u)g(t-u)\,du\right]$
$= F(s)G(s)$

ix) INITIAL VALUE

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

x) FINAL VALUE

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

FOURIER TRANSFORM

FOURIER SINE TRANSFORM

$F_s(\lambda) = \int_0^\infty f(u)\sin\lambda u\,du$

INVERSE FOURIER SINE

$f(x) = \frac{2}{\pi}\int_0^\infty F_s(\lambda)\sin\lambda x\,d\lambda$

FOURIER COSINE TRANSFORM

$F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\,du$

INVERSE COSINE FOURIER

$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\lambda)\cos\lambda x\,d\lambda$

INVERSE LAPLACE TRANSFORM

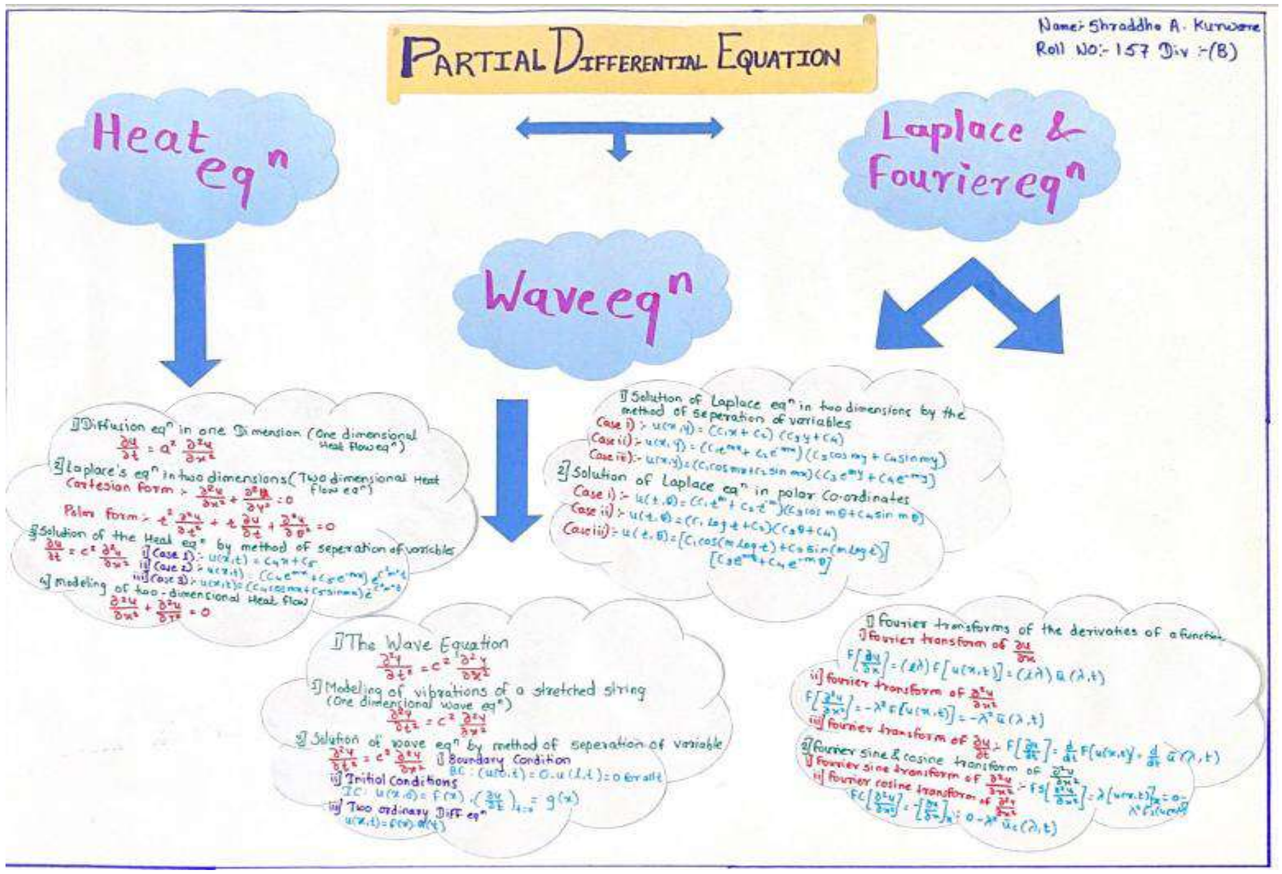
FOR ELEMENTARY FUNCTIONS

$\frac{1}{s}$	1	
$\frac{1}{s-a}$	e^{-at}	
$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$	
$\frac{s}{s^2+a^2}$	$\cos at$	
$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$	
$\frac{s}{s^2-a^2}$	$\cosh at$	
$\frac{1}{s^{n+1}}$	$\frac{t^n}{\lceil n+1}$	$\frac{t^n}{n!}$

GENERAL THEOREMS

INVERSE OF LAPLACE

GENERAL TRANFORMS



PARTIAL DIFFERENTIAL EQUATION

Name:- Shraddha A. Kurware
Roll No:- 157 Div :- (B)

Heat eq^n

I] Diffusion eq^n in one Dimension (One dimensional Heat Flow eq^n)

$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$

2] Laplace's eq^n in two dimensions (Two dimensional Heat Flow eq^n)

Cartesian Form :- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Polar Form :- $r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

3] Solution of the Heat eq^n by method of seperation of variables

$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$

i] Case 1) :- $u(x,t) = c_4x + c_5$

ii] Case 2) :- $u(x,t) = (c_4e^{mx} + c_5e^{-mx})e^{c^2m^2t}$

iii] Case 3) :- $u(x,t) = (c_4\cos mx + c_5\sin mx)e^{-c^2m^2t}$

4] Modeling of two-dimensional Heat flow

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Wave eq^n

I] The Wave Equation

$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$

2] Modeling of vibrations of a stretched string (One dimensional wave eq^n)

$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$

3] Solution of wave eq^n by method of seperation of variable

$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$

i] Boundary Condition

BC : $u(0,t) = 0,\ u(l,t) = 0$ for all t

ii] Initial Conditions

IC : $u(x,0) = f(x),\ \left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x)$

iii] Two ordinary Diff eq^n

$u(x,t) = F(x)\cdot G(t)$

Laplace & Fourier eq^n

I] Solution of Laplace eq^n in two dimensions by the method of seperation of variables

Case i) :- $u(x,y) = (c_1x + c_2)(c_3y + c_4)$

Case ii) :- $u(x,y) = (c_1e^{mx} + c_2e^{-mx})(c_3\cos my + c_4\sin my)$

Case iii) :- $u(x,y) = (c_1\cos mx + c_2\sin mx)(c_3e^{my} + c_4e^{-my})$

2] Solution of Laplace eq^n in polar Co-ordinates

Case i) :- $u(r,\theta) = (c_1r^m + c_2r^{-m})(c_3\cos m\theta + c_4\sin m\theta)$

Case ii) :- $u(r,\theta) = (c_1\log r + c_2)(c_3\theta + c_4)$

Case iii) :- $u(r,\theta) = [c_1\cos(m\log r) + c_2\sin(m\log r)][c_3e^{m\theta} + c_4e^{-m\theta}]$

I] Fourier transforms of the derivatives of a function

i] Fourier transform of $\frac{\partial u}{\partial x}$

$F\left[\frac{\partial u}{\partial x}\right] = (i\lambda)F[u(x,t)] = (i\lambda)\bar{u}(\lambda,t)$

ii] Fourier transform of $\frac{\partial^2 u}{\partial x^2}$

$F\left[\frac{\partial^2 u}{\partial x^2}\right] = -\lambda^2F[u(x,t)] = -\lambda^2\bar{u}(\lambda,t)$

iii] Fourier transform of $\frac{\partial u}{\partial t}$:- $F\left[\frac{\partial u}{\partial t}\right] = \frac{d}{dt}F[u(x,t)] = \frac{d}{dt}\bar{u}(\lambda,t)$

2] Fourier sine & cosine transform of $\frac{\partial^2 u}{\partial x^2}$

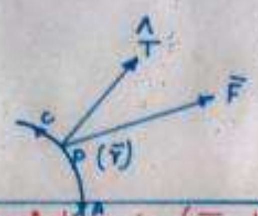
i] Fourier sine transform of $\frac{\partial^2 u}{\partial x^2}$:- $F_s\left[\frac{\partial^2 u}{\partial x^2}\right] = \lambda[u(x,t)]_{x=0} - \lambda^2F_s(u(x,t))$

ii] Fourier cosine transform of $\frac{\partial^2 u}{\partial x^2}$

$F_c\left[\frac{\partial^2 u}{\partial x^2}\right] = -\left[\frac{\partial u}{\partial x}\right]_{x=0} - \lambda^2\bar{u}_c(\lambda,t)$

Vector Integral Calculus and Applications

Appln.:
1) Differential geometry
2) Partial differential eqn
3) Physics and engineering.



A line integral = $\int_c \bar{F}.d\bar{r}$

$\int_c \bar{F} = d\bar{r} = \int_c F_1 dx + F_2 dy + F_3 dz$

Eg → Show that $\iiint_v \frac{dv}{r^2} = \iint_s \frac{\bar{r}.\hat{n}}{r^2} ds$.

By diverson theo^m

$$\iint_s \frac{\bar{r}.\hat{n} ds}{r^2} = \iiint_v \nabla.\left(\frac{\bar{r}}{r^2}\right) dv$$

$$\nabla.\left(\frac{\bar{r}}{r^2}\right) = \nabla.(\bar{r}.r^{-2}) = (\nabla.\bar{r})r^{-2} + \nabla(r^{-2}).\bar{r}$$
$$= \frac{3}{2} - 2r^{-4}\,\bar{r}.\bar{r} = \frac{3}{r^2} - \frac{2}{r^4}r^2 = \frac{1}{r^2}$$

$\iint_s \frac{\bar{r}.\hat{n}}{r^2} ds = \iiint_v \frac{dv}{r^2}$ ← result.

GAUSS - DIVERGENCE THEOREM.

It is written as

$$\iint_s \bar{F}.\hat{n} ds = \iiint_v \nabla.\bar{F} dv$$

Surface integral

Component of F is given by $\bar{F}.\hat{n}$

$\int_s \bar{F}.\hat{n}\, ds$ or $\iint_s (\bar{F}.\hat{n}) ds$

STOKE'S THEOREM AND Expressed

s is the open surface to which $\hat{n}$ is unit outword drawn normal vector. $\bar{F}$ is is acting at p enclosed by element ds. Curve 'c' is the boundary of the surface.

$$\iint_s \hat{n}.\text{curl}\,\bar{F} ds = \oint_c \bar{F}.d\bar{r}$$

Appl^n of Vectors To Fluid Mechanics

The Velocity potential

$\bar{q} = -\nabla\phi$

Here this ϕ is called as Velocity potential.

Vorticity Vector

In flows where, $\nabla \times \bar{q} \neq 0$ the vector $\zeta = \nabla \times \bar{q}$ is called a Vorticity Vector

Eq^n of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Euler Eq^n of Motion

$\bar{q} = u\bar{i} + v\bar{j} + w\bar{k}$

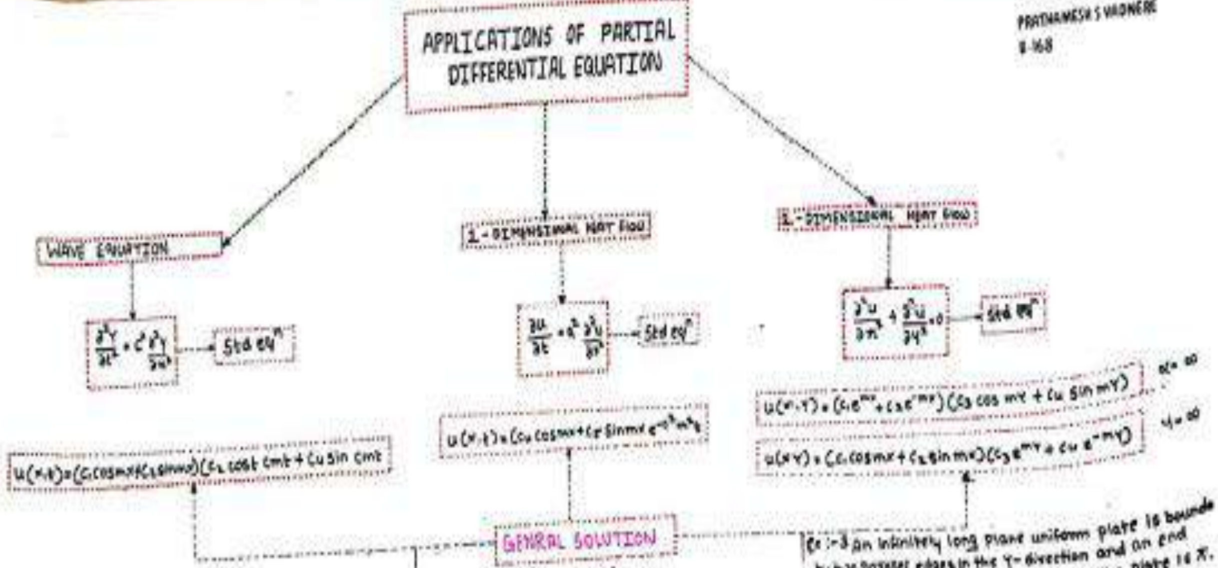
and $\bar{F} = F_x\bar{i} + F_y\bar{j} + F_z\bar{k}$

Integration of Euler's Eq^n of motio

$$\frac{1}{2}q^2 + v + \int \frac{dp}{\rho} = c$$

c = Constant of integration

PRATHAMESH S VADNERE
B-168



APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATION

WAVE EQUATION

$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ → Std eqⁿ

$u(x,t) = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt)$

1-DIMENSIONAL HEAT FLOW

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ → Std eqⁿ

$u(x,t) = (c_1 \cos mx + c_2 \sin mx) e^{-c^2 m^2 t}$

2-DIMENSIONAL HEAT FLOW

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ → Std eqⁿ

$u(x,y) = (c_1 e^{mx} + c_2 e^{-mx})(c_3 \cos my + c_4 \sin my)$ x = ∞

$u(x,y) = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my})$ y = ∞

GENRAL SOLUTION

Ex :- 1 : A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $u = a \sin \pi x/l$ from which it is released at time t = 0. Find the displacement u(x,t) from one end. [use wave eqⁿ $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$]

→ Given wave eqⁿ is $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

As the point of the string are fixed for all time and the initial transverse velocity of any point of the string is zero, we solve the wave eqⁿ.

① $u(0,t) = 0, \forall t$
② $u(l,t) = 0, \forall t$
③ $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$
④ $u(x,0) = a \sin \frac{\pi x}{l}$

The most general solⁿ is
$u(x,t) = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt)$

Condition (i) $u(0,t) = 0\ \forall t \Rightarrow \boxed{c_1 = 0}$

∴ solⁿ become
$u(x,t) = c_2 \sin mx (c_3 \cos cmt + c_4 \sin cmt)$

differentiating $\frac{\partial u}{\partial t} = c_2 \sin mx [c_3(-cm \sin cmt) + c_4 (cm \cos cmt)]$

condition (iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \Rightarrow \boxed{c_4 = 0}$

∴ solⁿ becomes
$u(x,t) = c_2 \sin mx (c_3 \cos cmt)$
$u(x,t) = c_5 \sin mx \cos cmt$

Applying condition (ii) $u(l,t) = 0, \forall t$
$0 = c_5 \sin ml \cos cmt$
$c_5 \neq 0$; $\cos cmt \neq 0$
∴ $\sin ml = 0$
$ml = n\pi$; $m = \frac{n\pi}{l}$, n = 1, 2, 3

∴ substituting in (1), we get
$u(x,t) = c_5 \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$, n = 1, 2, 3

combining these solⁿ we get.

Applying condition (iv) $u(x,0) = a \sin \frac{\pi x}{l}$

$a \sin \frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

$a \sin \frac{\pi x}{l} = b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + \dots$

$b_1 = a$, $b_2 = 0 = b_3 = \dots = b_n = \dots$

(2) will become
$u(x,t) = b_1 \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l} + b_2 \sin \frac{2\pi x}{l} \cos \frac{2\pi ct}{l} + \dots$

$\boxed{u(x,t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}}$

which is the required general solⁿ.

Ex 2 : Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ if,

i. u is finite for all t.
ii. $u(0,t) = 0, \forall t$
iii. $u(l,t) = 0, \forall t$
iv. $u(x,0) = u_0$ for $0 \le x \le l$, where l being the l of the bar.

→ The most general solⁿ is
$u(x,t) = (c_1 \cos mx + c_2 \sin mx) e^{-m^2 c^2 t}$

By (ii), $u(0,t) = 0\ \forall t \Rightarrow \boxed{c_1 = 0}$
$u(x,t) = c_2 \sin mx\, e^{-m^2 c^2 t}$

By (iii) $u(l,t) = 0, \forall t \Rightarrow 0 = c_2 \sin ml\, e^{-m^2 c^2 t}$
$\sin(ml) = 0$, $ml = n\pi$, n = 1, 2, 3
$m = \frac{n\pi}{l}$, n = 1, 2, 3
$u(x,t) = c_2 \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 c^2 t}{l^2}}$, n = 1, 2

∴ $u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 c^2 t}{l^2}}$

By (iv) applying condition $u(x,0) = u_0$ for $0 \le x \le l$

$u_0 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

$b_n = \frac{2}{l} \int_0^l u_0 \sin \frac{n\pi x}{l} dx = \frac{2u_0}{l} \left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right)_0^l$

$b_n = \frac{2u_0}{\pi} \left[\frac{1-(-1)^n}{n} \right] = \begin{cases} 0 & \text{if } n \text{ even} \\ \frac{4u_0}{n\pi} & \text{if } n \text{ odd} \end{cases}$ Put n = 2n+1, n = 0, 1, 2 ...

$u(x,t) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \left(\frac{1-(-1)^n}{n} \right) \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 c^2 t}{l^2}}$

$\boxed{\therefore u(x,t) = \frac{4u_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin \frac{(2n+1)\pi x}{l} e^{-\frac{(2n+1)^2 \pi^2 c^2 t}{l^2}}}$

Ex :- 3 An infinitely long plane uniform plate is bounded by two parallel edges in the y-direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at temp u_0 at all points and other edges at zero temp. Find the steady-state temp function u(x,y).

→ Solⁿ :- we have to solve the P.D.E $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the condition
① $u(0,y) = 0$
② $u(\pi,y) = 0$
③ $u(x,\infty) = 0$ for $0 < x < \pi$
④ $u(x,0) = u_0$ for $0 < x < \pi$

The most general solⁿ is
$u(x,y) = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my})$

(iii) ⇒ $\boxed{c_3 = 0}$
(i) ⇒ $\boxed{c_1 = 0}$

$u(x,y) = c_5 \sin mx\, e^{-my}$
$0 = c_5 \sin m\pi\, e^{-my}$
$c_5 \neq 0$, $e^{-my} \neq 0$, $\sin m\pi = 0$
$m\pi = n\pi$; $m = n$
$u(x,y) = c_5 \sin nx\, e^{-ny}$

or $u(x,y) = \sum_{n=1}^{\infty} b_n \sin nx\, e^{-ny}$

condition (iv) is
$u_0 = \sum_{n=1}^{\infty} b_n \sin nx$ $0 < x < \pi$

$b_n = \frac{2}{\pi} \int_0^{\pi} u_0 \sin nx\, dx = \frac{2u_0}{\pi} \left(\frac{-\cos nx}{n} \right)_0^{\pi} = \frac{2u_0}{\pi} \left(\frac{1-(-1)^n}{n} \right)$

∴ $u(x,y) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \left(\frac{1-(-1)^n}{n} \right) \sin nx\, e^{-ny}$

$\boxed{u(x,y) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x\, e^{-(2n-1)y}}$

Roll No : 94

Shortcut Method

Case I

PI when $f(x) = \begin{cases} e^{ax} \ldots \text{put } D = a \\ a^{x} \ldots \text{put } D = \log a \\ k \ldots \text{put } D = 0 \end{cases}$

Case II

PI when $f(x) = \begin{cases} \sin(ax+b) \ldots \text{put } D^2 = -a^2 \\ \cos(ax+b) \ldots \text{put } \frac{1}{D} = \int \end{cases}$

Case III

PI when $f(x) = \begin{cases} \sinh(ax+b) \ldots \text{put } D^2 = -a^2 \\ \cosh(ax+b) \ldots \text{put } \frac{1}{D} = \int \end{cases}$

Case IV

PI when $f(x) = x^m$ ... Use Binomial series.

Case V

PI when $f(x) = e^{ax} V$ ... Take e^{ax} OUT put $D = D + a$

Case VI

PI when $f(x) = \begin{cases} x^m \cos ax \\ x^m \sin ax \end{cases}$

Case VII

PI when $f(x) = xV$

$$\frac{1}{F(D)} xV = \left[x - \frac{f'(D)}{F(D)}\right] \frac{1}{f(D)} V$$

General Method

$$\frac{1}{D+M} f(x) = e^{-mx} \int e^{mx} f(x)\, dx$$

$$\frac{1}{D-M} f(x) = e^{mx} \int e^{-mx} f(x)\, dx$$

$$\frac{1}{D} f(x) = \int f(x)\, dx$$

Method of Variation of parameters

$f(D)y = X$

$y_c = c_1 y_1 + c_2 y_2$

$y_p = u y_1 + v y_2$

where, $u = -\int \frac{y_2 X}{W} dx$

$v = \int \frac{y_1 X}{W} dx$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

Real & distinct $= c_1 e^{m_1 x} + c_2 e^{m_2 x}$

Real & repeated $= (c_1 x + c_2) e^{m_1 x}$

complex $= e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

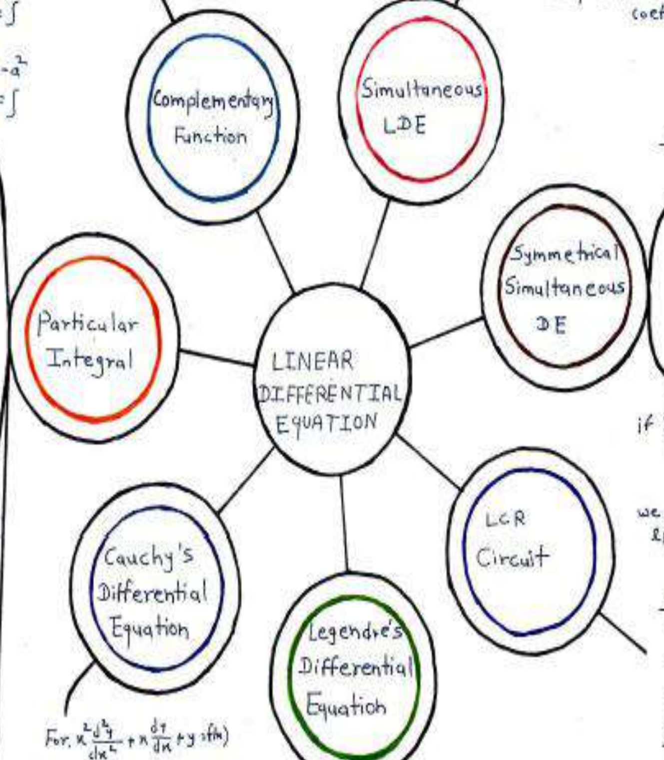
complex & repeated $= e^{\alpha x}[(c_1 x + c_2)\cos \beta x + (c_3 x + c_4) \sin \beta x]$

$\frac{dx}{dt} + y = t$, $\frac{dy}{dt} - x = e^{t}$

step 1 : put $D = \frac{d}{dt}$

step 2 : Solve for x Using cramer's rule

step 3 : Use the equation where the coefficient of y is simple.



Variable Separable form

$\frac{dx}{y} = \frac{dy}{x}$

$x\,dx = y\,dy$

$\int x\,dx = \int y\,dy$

$x^2/2 = y^2/2 + c_1$

$x^2/2 - y^2/2 = c_1$

$x^2 - y^2 = c$

Method of Multipliers

if $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR} = \frac{l_1 dx + m_1 dy + n_1 dz}{l_1 P + m_1 Q + n_1 R}$

we choose l, m, n in such way that

$lP + mQ + nR = 0$ & $l_1 P + m_1 Q + n_1 R = 0$

then $u(x,y,z) = c_1$; $v(x,y,z) = c_2$

case 1 : The differential equation of electrical circuit consists of L , C with emf E is

$L \frac{dI}{dt} + \frac{Q}{C} = E \quad \therefore L \frac{d^2 Q}{dt^2} + \frac{Q}{C} = E$

case 2 : The differential equation of electrical circuit consists of L, C

$L \frac{d^2 Q}{dt^2} + \frac{Q}{C} = 0 \quad \therefore \frac{d^2 Q}{dt^2} + \frac{Q}{LC} = 0$

case 3 : The DE of circuit consists of L, R & C with E $\frac{d^2 Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{E}{L}$

case 4 : The DE consists of L, R, & C without E is $\frac{d^2 Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$

For, $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = f(x)$

put, $x = e^{z}$

$\log x = z$

$\frac{d}{dz} = D$

$x \frac{dy}{dx} = Dy$

$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$

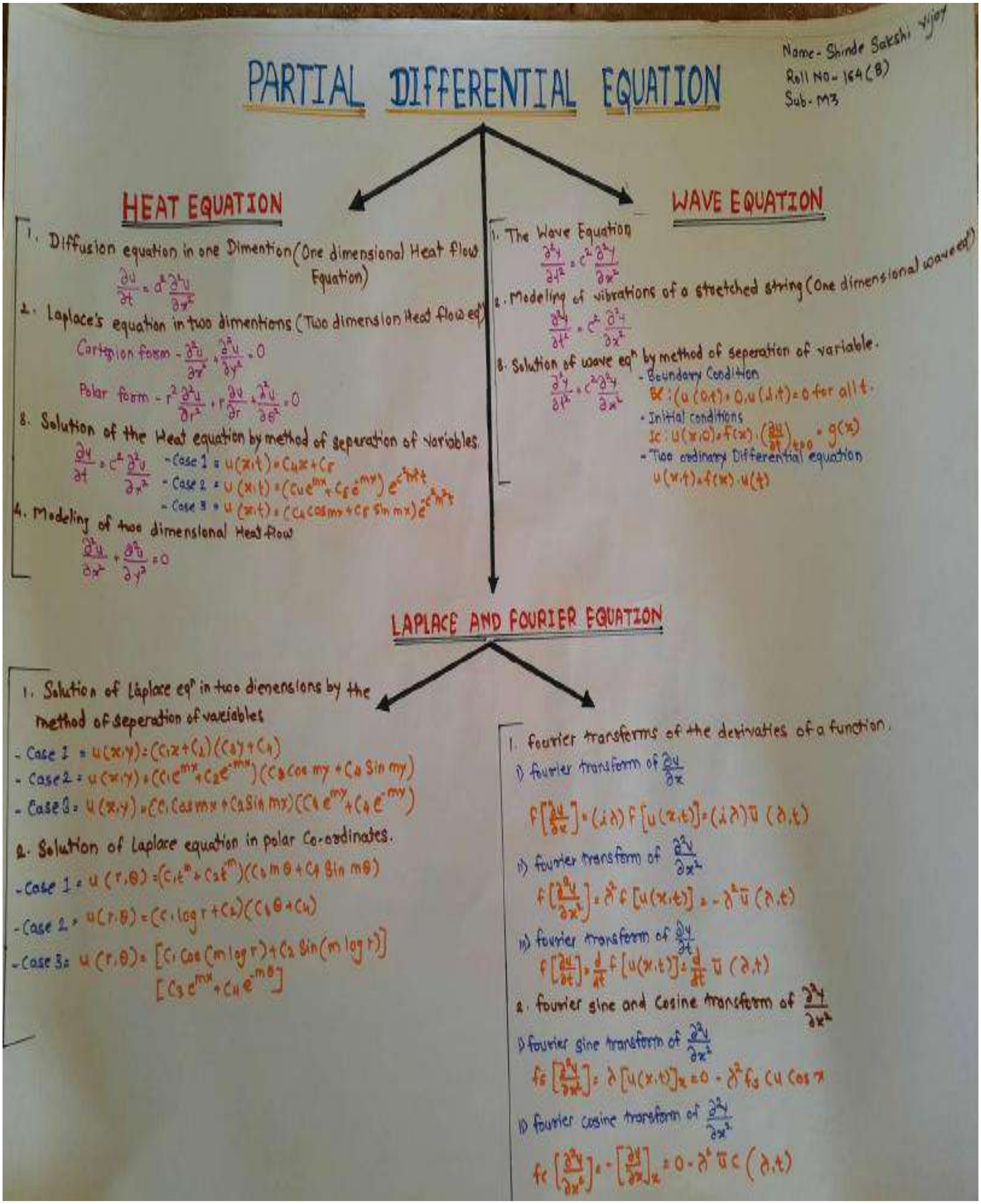
For, $(ax+b)^2 \frac{d^2 y}{dx^2} + (ax+b)\frac{dy}{dx} + y = f(x)$

put $(ax+b) = e^{z}$, $\log(ax+b) = z$

$\frac{d}{dz} = D$

$(ax+b)\frac{dy}{dx} = aDy$

$(ax+b)^2 \frac{d^2 y}{dx^2} = (a)^2$



Name- Shinde Sakshi Vijay
Roll NO- 164 (B)
Sub- M3

PARTIAL DIFFERENTIAL EQUATION

HEAT EQUATION

1. Diffusion equation in one Dimention (One dimensional Heat flow Equation)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

2. Laplace's equation in two dimentions (Two dimension Heat flow eq^n)

Cartesion form - $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Polar form - $r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

3. Solution of the Heat equation by method of separation of variables.

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- Case 1 = $u(x,t) = C_4 x + C_5$
- Case 2 = $u(x,t) = (C_4 e^{mx} + C_5 e^{-mx}) e^{c^2 m^2 t}$
- Case 3 = $u(x,t) = (C_4 \cos mx + C_5 \sin mx) e^{-c^2 m^2 t}$

4. Modeling of two dimensional Heat flow

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

WAVE EQUATION

1. The Wave Equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

2. Modeling of vibrations of a stretched string (One dimensional wave eq^n)

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

3. Solution of wave eq^n by method of seperation of variable.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

- Boundary Condition
  BC : $(u(0,t) = 0, u(l,t) = 0$ for all t.
- Initial conditions
  IC : $u(x,0) = f(x), \left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x)$
- Two ordinary Differential equation
  $u(x,t) = f(x) \cdot u(t)$

LAPLACE AND FOURIER EQUATION

1. Solution of Laplace eq^n in two dienensions by the method of seperation of variables

- Case 1 = $u(x,y) = (C_1 x + C_2)(C_3 y + C_4)$
- Case 2 = $u(x,y) = (C_1 e^{mx} + C_2 e^{-mx})(C_3 \cos my + C_4 \sin my)$
- Case 3 = $u(x,y) = (C_1 \cos mx + C_2 \sin mx)(C_3 e^{my} + C_4 e^{-my})$

2. Solution of Laplace equation in polar Co-ordinates.

- Case 1 = $u(r,\theta) = (C_1 r^m + C_2 r^{-m})(C_3 m\theta + C_4 \sin m\theta)$
- Case 2 = $u(r,\theta) = (C_1 \log r + C_2)(C_3 \theta + C_4)$
- Case 3 = $u(r,\theta) = [C_1 \cos(m \log r) + C_2 \sin(m \log r)][C_3 e^{mx} + C_4 e^{-m\theta}]$

1. fourier transferms of the derivaties of a function.

i) fourier transform of $\frac{\partial u}{\partial x}$

$$F\left[\frac{\partial u}{\partial x}\right] = (i\lambda) F[u(x,t)] = (i\lambda) \bar{u}(\lambda,t)$$

ii) fourier transform of $\frac{\partial^2 u}{\partial x^2}$

$$F\left[\frac{\partial^2 u}{\partial x^2}\right] = \lambda^2 F[u(x,t)] = -\lambda^2 \bar{u}(\lambda,t)$$

iii) fourier transform of $\frac{\partial u}{\partial t}$

$$F\left[\frac{\partial u}{\partial t}\right] = \frac{d}{dt} F[u(x,t)] = \frac{d}{dt} \bar{u}(\lambda,t)$$

2. fourier sine and Cosine transform of $\frac{\partial^2 u}{\partial x^2}$

i) fourier sine transform of $\frac{\partial^2 u}{\partial x^2}$

$$f_s\left[\frac{\partial^2 u}{\partial x^2}\right] = \lambda [u(x,t)]_{x=0} - \lambda^2 f_s (u \cos x$$

ii) fourier cosine transform of $\frac{\partial^2 u}{\partial x^2}$

$$f_c\left[\frac{\partial^2 u}{\partial x^2}\right] = -\left[\frac{\partial u}{\partial x}\right]_x = 0 - \lambda^2 \bar{u}_c(\lambda,t)$$

13. For review and critique contact: e-mail address of faculty and HOD

dhumase.manisha@kbtcoe.org **hod.mech@kbtcoe.org**

Dhumase

Ms. M.G.Dhumase
Subject Incharge

Dr. S. M. Bhati
Module Coordinator

Dr.V.C.Shewale
HOD